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Sensitivity of a Plate Pyroelectric Detector to Ambient Acoustic Noise: The Significance of the Perfectly-Clamped Mounting Condition*

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The unwanted sensitivity of a plate pyroelectric detector to airborne acoustic noise is depends critically on the mounting conditions for the plate. We consider a plate in an isotropic acoustic pressure field oscillating at angular frequency ω_d. *We analyze the situation in terms of a mechanically isotropic material, and show that for one type of boundary conditions, the so-called clamped-boundary conditions, the strain-induced time-varying surface charges caused by the pressure in various regions of the plate, completely cancel out. This suggests that significantly reduced noise sensitivity of a practical free-standing pyroelectric detector can be achieved by paying careful attention to the mounting conditions. We also derive the strains for the case of the trigonal 3m class within the framework developed for the thin-plate approximation, and show that the result is unchanged. This has implications for the design of pyroelectric detectors based on a stiff material such as* $LiTaO_3$.

Keywords Acoustic noise; anisotropy; domain engineering; ferroelectric; mounting conditions; piezoelectric; plate deformation; pyroelectric detector; strain

Background

The study of pyroelectric detectors provides a unique challenge because the electrical, mechanical, and thermal properties are coupled to each other [1]. Knowledge of the thermal behaviour of a pyroelectric detector has directed us to building detectors that are thermally isolated (hence, freestanding) and as thin as possible for a given area [2]. The constraints that lead to greater thermal sensitivity, unfortunately, lead to undesirable trade-offs such as greater susceptibility to ambient acoustic noise [3]. Therefore we have derived a means of calculating a contribution to noise voltage from a freestanding pyroelectric disk. This includes piezoelectric noise generated by ambient acoustically excited flexural vibration, which can dominate all other noise sources-even in a quiet laboratory environment.

The present work presents analysis that advances our understanding of the behaviour of ferroelectric materials and pyroelectric detectors, and provides engineering tools for future

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development. We do not present experimental results in detail here, but rather investigate by approximate analytical methods the origins and possible methods of amelioration of this important effect. Our analysis is directed towards flexural acoustic modes in the low kilohertz range where ambient air-borne acoustic noise will be efficiently coupled to the detector plate.

In the past we have experimentally investigated the effects of ferroelectric domain reversal [4]. Using this technique, the induced surface charge in response to an acoustically induced strain may be caused to change sign. Correct balancing of natural and domain-inverted regions gives us another tool with which to reduce the sensitivity to acoustic noise without resorting to the mounting of the detector on a rigid substrate. Our analysis provides an objective basis for evaluating past and future experimental endeavours.

With knowledge of the sound pressure and the mechanical and material properties of the plate, we have first to calculate the plate deformation as a function of the plate geometry and thickness. From knowledge of the plate displacement, we quantify the strain as a function of position, which is necessary to calculate the electric displacement and surface charge density. The total charge generated by flexure is then obtained by integrating the surface charge density over the area of the detector covered by the electrode.

Introduction

The results of this paper are derived first for a plate of isotropic material, within the thin plate approximation (that is, plate thicknes $\ll$ acoustic wavelength). It is of course possible to construct a plate pyroelectric detector out of such material, and the results of this paper apply provided the thin plate condition is satisfied, which will always be the case with practical devices. To be conservative, and because there is a measure of ambiguity in formulating Poisson's ratio, we generalise the thin-plate approximation to the 3m case, starting from first principles, and find that the results are valid. We assert that a z-cut plate of 3m material, for example $LiTaO_3$, can be described with accuracy sufficient that the results of this paper are applicable. Later in this paper we present the strains in a thin plate of z-cut 3m material in terms of the z-directed displacement field $\zeta(x, y)$ and the elastic coefficients, defining along the way the quantities $\lambda = c_{14}/c_{44}$ and $\mu = c_{13}/c_{33}$. The transition from the 3m case (6 independent stiffness coefficients: $c_{11}, c_{12}, c_{13}, c_{14}, c_{33}, c_{44}$) case to the lower-symmetry isotropic case (2 independent stiffness coefficients: c_{11}, c_{12}) is accomplished by letting $c_{14} \to 0, c_{33} \to c_{11}, c_{13} \to c_{12}$, and $c_{44} \to (c_{11} - c_{12})/2$, implying that $\lambda \to 0$ and $\mu \to c_{12}/c_{11} = \nu/(1-\nu)$, where ν is Poisson's ratio.

The damped wave equation for a thin isotropic plate has the form

$$c^4\nabla^4 u + 2\gamma\frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial t^2} = Z(\underline{r}, t), \tag{1}$$

where

$$c^4 = \frac{Eh^2}{3\rho(1-\nu^2)}, \tag{2}$$

γ is the magnitude of damping, u is the plate displacement, E is the modulus of elasticity (or Young's modulus for an isotropic plate), $2h$ is the plate thickness, ρ is the density of the plate material, t is time, and Z is the magnitude of the driving oscillator proportional to the sound pressure.

It is possible to solve Eq. (1) by expanding the amplitude u and the driving term Z in terms of the eigenfunctions of the associated undamped, undriven wave equation, which is

$$c^4\nabla^4 u_n^N - \omega_n^2 u_n^N = 0, \tag{3}$$

where ω_n is the angular frequency of the nth eigenmode. The general displacement may be expanded as

$$u(\underline{r}, t) = \sum_{n,n'} a_{n,n'}(t) u_{n,n'}^N(\underline{r}). \tag{4}$$

The expansion of the driving term on the right side of Eq. (1) is

$$Z = \sum_{m,m'} b_{m,m'}(t) u_{m,m'}^N(\underline{r}) e^{i\omega_d t}, \tag{5}$$

where $\underline{r}$ is a transverse vector in the plane of the plate. We assume a deterministic and harmonic driving term for the present analysis. In order to solve Eq. (3), the physical manifestation of the plate boundary conditions, that is, clamped, simply supported, or some other variation, must be established at this stage. For the case of a circular disk with radius $r = a$ clamped about its perimeter, we require

$$u_n(a, \theta) = 0, \tag{6}$$

and

$$\frac{\partial u_n(a, \theta)}{\partial r} = 0, \tag{7}$$

which is the mathematical representation of the clamped boundary conditions. The eigenvalue equation is now

$$\frac{J_n(ka)}{I_n(ka)} + \frac{J_n'(ka)}{I_n'(ka)} = 0, \tag{8}$$

where I_n, I_n' and J_n, J_n' are Bessel functions. If we denote the mth root of Eq. (8) by $Z_{m,n}$, then

$$z_{m,n} = k_{m,n} a, \tag{9}$$

and the normalised eigenfunctions are of the form

$$u_{n,m}^N(r, \theta) = u_{n,m} \cos n\theta \left\{ J_n\left(\frac{z_m r}{a}\right) - \frac{J_n(z_m)}{I_n(z_m)} I_n\left(\frac{z_m r}{a}\right) \right\}. \tag{10}$$

Equation (10) is valid even if the boundary conditions are altered. If they are altered, then the eigenvalue equation described by Eq. (9) will no longer apply. Classically there are two extreme cases of boundary conditions for the vibrating plate: perfectly clamped, and simply supported. These two conditions are illustrated in cross section in Fig. 1 (a) and (b).

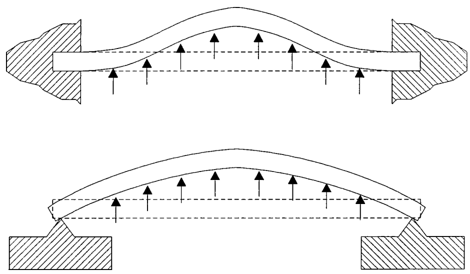

FIGURE 1 Two idealized plate boundary conditions in the context of physical mounting and deformation: (a) clamped; (b) simply supported.

The Boundary Conditions—A Brief Discussion

It is not obvious that the flexural displacement of a ferroelectric plate that is clamped by peripheral rings obeys either the clamped (Fig. 1(a)) or the simply supported (Fig. 1(b)) boundary condition, because the displacements near the perimeter are small (of the order of nanometers or less). This difference is more readily indicated by resonant frequencies predicted by Eq. (8) for the clamped condition and those for the solutions of the eigenvalue equation appropriate to the simply supported case. It is useful to have some theory of the boundary conditions in which a parameter is used to describe a smooth interpolation between the two cases (a) and (b) of Fig. 1. Kantham [5] has described such a generalization of the situation, where a single parameter allows the boundary conditions to smoothly evolve between these extreme cases, the free plate being irrelevant in this present discussion.

Kantham [5] defines an additional boundary condition expression for an elastically restrained plate following Timoshenko [6]: $\alpha = \beta M$, where α is the slope at the plate perimeter, M is the turning moment per unit length at the plate perimeter, and β is the *elastic restraint factor*. The elastic boundary condition may be written within Kantham's generalization as

$$\left(\frac{\partial u}{\partial r}\right)_{r=a} = -\beta R_{iso}\left\{\frac{\partial^2 u}{\partial r^2} + \frac{\nu}{r}\frac{\partial u}{\partial r}\right\}_{r=a}, \tag{11}$$

where the modulus of rigidity R_{iso} is given in the isotropic case by

$$R_{iso} = c^4 \rho 2h \tag{12}$$

and the M, turning moment per unit length of perimeter, is given by R_{iso} multiplied by the term in curly brackets on the right hand side of Eq. (11). The eigenvalue equation now has the form [7] of

$$-\left\{\frac{J_{n+1}(z_n a)}{J_n(z_n a)} + \frac{I_{n+1}(z_n a)}{I_n(z_n a)}\right\}\left(1 - \frac{\beta R_{iso}}{a}(1-\nu)\right) = 2\frac{\beta R_{iso}}{a}(z_n a), \tag{13}$$

the limits of which are bounded between the simply supported case, $\beta \to \infty$, where there is no resistance to the turning moment, and the perfectly clamped case, $\beta \to 0$, where a very large turning moment is needed to produce even the smallest radial gradient near the perimeter. Equation (11) or alternatively, Eq. (13), reduces to Eq. (8) for the perfectly clamped case. For the simply supported case, imposing the condition $\beta \to \infty$ ensures that additionally, the term in the curly brackets on the right of Eq. (11) must be zero. The simultaneous vanishing of the radial gradient at the boundary and the right hand side of Eq. (11) are the required conditions for the simply supported case.

It is important to note that Eq. (13) can be generalized to apply to the lower symmetry 3m case appropriate to a $LiTaO_3$ plate. The resulting equation is similar to Eq. (13) but the constant R_{iso} is replaced by a more complicated combination of the stiffness coefficients.

If the plate comprising the pyroelectric detector is set in oscillation by means of ambient acoustic vibration, then it is interesting to consider how the spurious piezoelectrically generated noise depends on the boundary conditions. If one takes a snapshot in time of the vibrating plate, then for symmetrical (that is, θ-independent) vibrations, one finds that there is a radius where the piezoelectrically induced surface charge changes polarity (assuming that the electrodes have negligible mass and cover each face entirely). This radius depends critically on the boundary conditions of the plate. What is surprising is that if the boundary conditions can be controlled so that the plate is effectively clamped, the quantities of positive and negative surface charge balance. This indicates that if one could mount the detector plate so that the clamped boundary conditions were obeyed, a great reduction (theoretically complete elimination) of sensitivity to ambient acoustic noise would be achieved. The only requirement for cancellation is that $u^N_{n,m}(a, \theta) = 0$, that is, that the displacement vanishes at $r = a$. Thus we can interpret Eq. (10) as the correct form for the displacement of any mode on the disk, provided that we get the value of z_m from the correct boundary-value equation. Furthermore, if the boundary conditions (not perfectly clamped) are predictable, in principle one could use ferroelectric domain engineering[4] to balance out the effects and arrive at a situation where the net charge collected by the surface electrode is zero.

We consider the excitation of the plate by means of ambient sound; take for simplicity the case where a single frequency excitation is occurring. The sound field is isotropic and wavelengths are long compared to typical detector dimensions, so from here on we take $n = 0$ to correspond with the assumption that there is no angular dependence in the ambient sound field. The normalized functions now

$$u^N_{0,m}(r, \theta) = u_{0,m}\left\{ J_n\left(\frac{z_m r}{a}\right) - \frac{J_n(z_m)}{I_n(z_m)} I_n\left(\frac{z_m r}{a}\right)\right\} \tag{14}$$

and

$$u_{0,m} = \frac{1}{\sqrt{2\pi}\, a\, J_0(z_m)}. \tag{15}$$

Displacement, Strain, and Charge Cancellation

The general displacement equation is

$$u(\underline{r}, t) = \sum_{n=1}^{\infty} a_n(t) u^N_n(r), \tag{16}$$

where

$$u^N_n(r) = \frac{1}{a\sqrt{2\pi}} \left\{ \frac{J_0\left(\frac{z_n r}{a}\right)}{J_0(z_n)} - \frac{I_0\left(\frac{z_n r}{a}\right)}{I_0(z_n)} \right\}, \tag{17}$$

where z_n is the nth root of the appropriate eigenvalue equation. Thus

$$b_n = \left(\frac{P}{2h\rho}\right)\frac{\sqrt{8\pi}a}{z_n}\frac{J_1(z_n)}{J_0(z_n)}, \tag{18}$$

where P is the peak air pressure, ρ is the plate density, $2h$ is the plate thickness, ω_d is the driving frequency of the sound pressure, and the resonant frequencies ω_n are given by

$$c^4 = \frac{Eh^2}{3\rho(1-\sigma^2)}, \tag{19}$$

$$z_n = k_n a, \tag{20}$$

and

$$\omega_n = z_n^2 \frac{h}{a^2}\sqrt{\left(\frac{E}{3\rho(1-\nu^2)}\right)}. \tag{21}$$

From Landau and Lifshitz [8] the nonzero strains written in terms of the z-directed displacement field $\zeta(x, y) = u(x, y, z = 0)$ are

$$\begin{aligned} S_1 &= -z\zeta_{,xx} \\ S_2 &= -z\zeta_{,yy} \\ S_3 &= \frac{\nu}{1-\nu} z(\zeta_{,xx} + \zeta_{,yy}) \\ S_6 &= -z\zeta_{,xy}. \end{aligned} \tag{22}$$

The strains in Eq. (22) are written in shorthand notation for the second partial derivatives; for example, $\zeta_{,xx} = \partial^2\zeta(r)/\partial x^2$, where the radial coordinate r is in a plane defined by orthogonal coordinates x and y. We want to determine the electric displacement vector, D_i and hence the displacement current within the framework of the thin-plate approximation. The appropriate piezoelectric constitutive equation is

$$D_i = \varepsilon_{ij}^E E_j - e_{iI} S_I, \tag{23}$$

where ε_{ij}^E is the constant-field permittivity, E_j is the electric field vector, e_{iI} is the piezoelectric stress, and S_I is the strain (this expression combines tensor and matrix notation for brevity [9]). In the present case, we consider the output of the pyroelectric to be connected into a low-impedance amplifier. Therefore we may consider the electric field to be zero throughout the thickness of the plate and the instantaneous voltage $V = (2h)(E_3) = 0$ is valid within the thin plate approximation and, again, within the framework of the thin-plate model.

We concern ourselves only with the z-direction component of the electric displacement in order to determine the surface charge density, so expanding, Eq. (23) and combining terms gives

$$D_3 = e_{31}(S_1 + S_2) + e_{33}S_3. \tag{24}$$

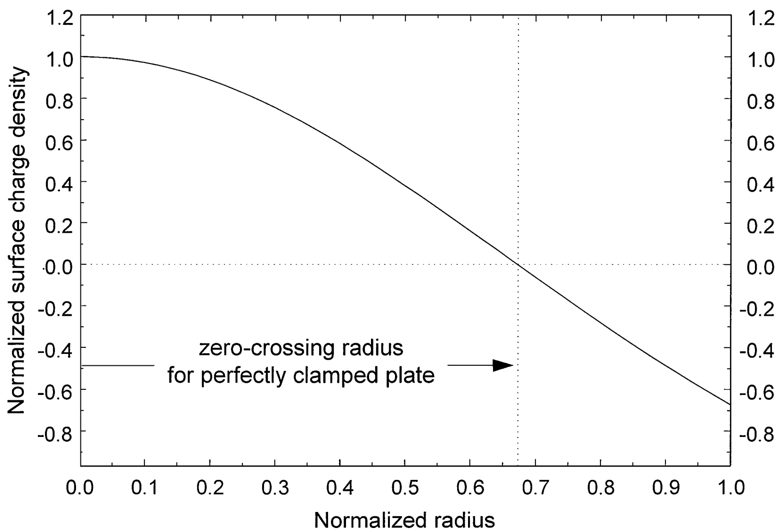

FIGURE 2 Calculated surface-charge density for a clamped freestanding circular plate.

Further expansion in terms of the actual displacement values as shown in Eq. (22) yields

$$D_3 = z\left[\frac{\partial^2\zeta(r)}{\partial x^2} + \frac{\partial^2\zeta(r)}{\partial y^2}\right]\left[\frac{\nu}{1-\nu}e_{33} - e_{31}\right]. \tag{25}$$

Thus at the surface of the disk, the instantaneous surface charge is

$$\sigma_s = \frac{h}{1-\nu}\left[\frac{\partial^2\zeta(r)}{\partial x^2} + \frac{\partial^2\zeta(r)}{\partial y^2}\right][\nu(e_{33} + e_{31}) - e_{31}], \tag{26}$$

with a corresponding negative value on the lower surface. A normalized plot of the instantaneous surface charge in Eq. (26) is shown in Fig. 2. Note that Eq. (26) now has the form of a divergence multiplied by a constant, or

$$\sigma_s = \nabla^2\zeta(r) \times K, \tag{27}$$

where K is a constant. In order to determine the total current generated by the disk, it is necessary to integrate the charge over the electrode area (for the present we assume the electrodes completely cover each face of the detector). Gauss's theorem transforms the divergence by

$$\int_{disk\ area} \nabla^2 f(r)dA = \oint_{disk\ perimeter} [\nabla f(r)] \cdot \underline{\hat{n}}d\ell, \tag{28}$$

where

$$\nabla \equiv \underline{\hat{r}}\frac{\partial}{\partial r} + \underline{\hat{\theta}}\frac{1}{r}\frac{\partial}{\partial\theta} \equiv \underline{\hat{r}}\frac{\partial}{\partial r}, \tag{29}$$

and so we can express the instantaneous surface charge in terms of the instantaneous gradient at the perimeter. From Eq. (29), in the case of clamped boundary conditions both the radial and angular derivatives of displacement tend to zero, and so the net instantaneous charge on the circular plate is zero. We now show that this key result holds not only for materials of the less symmetric 6 mm crystal class but also for 3m-class materials, again all within the framework of the thin-plate approximation.

Anisotropy and the Treatment of the 3m Crystal Class

As shown before, for the isotropic case, we find that D_3 is proportional to a divergence. An analysis of the free energy of a strained 3m thin plate (electrodes covering the z-faces and electrically short circuited) whose neutral surface intitially coincides with the $z = 0$ plane, shows that the strains can be expressed in terms of the displacement of the neutral surface $\zeta(x, y)$ as follows:

$$\begin{aligned} S_1 &= -z\zeta_{,xx}; \\ S_2 &= -z\zeta_{,yy}; \\ S_3 &= \mu z(\zeta_{,xx} + \zeta_{,yy}); \\ S_4 &= \lambda z(\zeta_{,xx} - \zeta_{,yy}); \\ S_5 &= 2\lambda z(\zeta_{,xy}); \\ S_6 &= -2z(\zeta_{,xy}). \end{aligned} \tag{30}$$

For convenience we have defined

$$\lambda = \frac{c_{14}}{c_{44}}; \quad \mu = \frac{c_{13}}{c_{33}}. \tag{31}$$

To compare, we refer to the isotropic case described by Eq. (22), where only four of the six possible strains are nonzero.

Applying again the piezoelectric stress tensor appropriate to the 3m class of crystals (in matrix notation) and the simplified constitutive relation (23) for electric displacement to find the component of D_i normal to the z-plane,

$$D_3 = e_{31}(S_1 + S_2) + e_{33}S_3. \tag{32}$$

No other components of the strain or piezoelectric tensor are involved. Expanding Eq. (32) by use of Eq. (30),

$$D_3 = (e_{33}\mu - e_{31})z\{u_{,xx} + u_{,yy}\}. \tag{33}$$

From Eq. (32) the charge density on the plate at the surface (at h) is

$$\sigma_s(x, y, h) = h(\mu e_{33} - e_{31})\{u_{,xx}(x, y) + u_{,yy}(x, y)\}. \tag{34}$$

Note that the c_{14} elastic component does not appear in the expressions for strains $S_1 S_2$ and S_3 in Eq. (31) and so the “extra” strains S_4 and S_5, which appear in the 3m but not the 6 mm class of materials, do not affect D_3 as given by Eq. (33). The latter equation is therefore valid for both the hexagonal 6 mm class of (transversely isotropic) materials and the trigonal

3m class of lower symmetry. The instantaneous charge on the flexing piezoelectric plate is given by integrating the surface charge of the plate area A so that

$$Q = \iint \sigma_s(x, y, h)dA. \tag{35}$$

The displacement current density j is given by the partial derivative of the electric displacement with respect to time. Therefore the displacement current density is

$$j_3(x, y, h) = h(\mu e_{33} - e_{31})\{\dot{u}_{,xx}(x, y) + \dot{u}_{,yy}(x, y)\}. \tag{36}$$

The time dependence of u is $e^{i\omega}d^t$, so

$$j_3(x, y, h) = h(i\omega_d)(\mu e_{33} = e_{31})\{u_{,xx}(x, y) + u_{,yy}(x, y)\}. \tag{37}$$

The total current generated $i_3(t)$ is obtained by integrating Eq. (37) over the area of the face of the disk, or

$$i_3(t) = h(i\omega_d)(\mu e_{33} - e_{31}) \iint_{disk\ area} \nabla^2 u(x, y)dA, \tag{38}$$

and using Gauss's theorem,

$$i_3(t) = h(i\omega_d)(\mu e_{33} - e_{31}) \oint_{\substack{disk\\perimeter}} \left[\frac{\partial u(x, y)}{\partial r}\right]_{r=a} d\ell. \tag{39}$$

In all cases we must use the appropriate eigenvalue equation to find z_n. If z_n is the solution to the clamped eigenvalue equation, the radial gradient $\partial u/\partial r$ at $r = a$ will vanish and so therefore will the acoustically generated noise current, according to Eq. (39). Following Kantham's treatment of the elastic boundary condition, $\partial u/\partial r$ at $r = a$ will not vanish for any other boundary condition (that is, any value of β other than zero) and so the instantaneous noise current i_3 will be proportional to the instantaneous radial gradient of the plate displacement, evaluated at the perimeter.

Finally, consider the θ-dependent case within the framework of the thin-plate approximation and in the context of Kantham's generalised boundary conditions. The question remains, if the theta-dependent flexural modes are in fact excited by the ambient acoustic field, will any of this theory still be valid? If we replace $f(r)$ in Eq. (28) by the most general form of solution $f(r)\cos(m\theta)$, we see that the theta part is irrelevant according to Eq. (29), $n \cdot \underline{\hat{\theta}} = 0$, and the amount of noise current is still determined by the radial gradient only. Thus, even if theta-dependent modes are excited, imposing the clamped-boundary conditions at $r = a$ will ensure that the noise current is zero.

Conclusions

We have presented a simple analytical model of a free-standing pyroelectric detector of circular boundary and argued that within the framework of the thin-plate approximation, the spurious acoustic noise generated through the piezoelectric effect by exposure to ambient acoustic noise depends critically on the exact nature of the clamping conditions of the plate, and becomes small (actually vanishes in our model) when the idealized clamped-boundary conditions are imposed, forcing the radial gradient of the plate displacement to zero at all times on the boundary. Our model assumes for simplicity a deterministic acoustic signal; a statistical treatment based on some assumed ambient noise power spectral density is expected to yield broadly the same interesting result but at the cost of greatly increased complexity. From this analysis we can assert that charge cancellation appears for the less-symmetric 3m case as it does for the nearly isotropic case.

References

1. S. B. Lang, *Sourcebook of Pyroelectricity* (Gordon and Breach, New York, 1974), pp. 167–375.
2. S. Bauer and B. Ploss, "A method for the measurement of the thermal, dielectric, and pyroelectric properties of thin pyroelectric films and their applications for integrated heat sensors," *J. Appl. Phys*. **68**, 6361–6367 (1990).
3. J. H. Lehman, A. M. Radojevic, R. M. Osgood, Jr., and M. Levy, "Fabrication and evaluation of a freestanding pyroelectric detector made from single-crystal $LiNbO_3$ film," *Opt. Lett.* **25**, 1657–1659 (2000).
4. J. H. Lehman and J. A. Aust, "Bicell pyroelectric optical detector made from a single $LiNbO_3$ domain-reveresed electret," *Appl. Opt*. **37**, 4210–4212 (1998).
5. C. Lakshmi Kantham, "Bending and vibration of elastically restrained circular plates," *J. Frank. Inst.* **256**, 483–491 (1958).
6. S. P. Timoshenko, *Theory of Plates and Shells* (McGraw-Hill, New York, 1940), p. 17.
7. C. Lakshmi Kantham, "Bending and vibration of elastically restrained circular plates," *J. Frank. Inst.* **256**, 483–491 (1958). Author's note: Equation (20) of Kantham has an incorrect negative sign on the right side. It is shown correctly in equation (13) of the present text.
8. L. D. Landau and E. M. Lifshitz, *Theory of Elasitcity*, 3rd Edition (Butterworth-Heinemann, Oxford, 1989), pp. 38–44.
9. J. F. Nye, *Physical Properties of Crystals* (Clarendon Press, Oxford, 1957), p. 134.